**PROJECT : CASH MANAGEMENT**

Saloni Singh -16ucs167

**SITUATION :**

Consider a national firm that receives checks from all over the United States.

Due to the vagaries of the U.S. Postal Service, as well as the banking system,

there is a variable delay from when the check is postmarked (and hence the

customer has met her obligation) and when the check clears (and when the

firm can use the money). For instance, a check mailed in Pittsburgh sent to

a Pittsburgh address might clear in just 2 days. A similar check sent to Los

Angeles might take 4 days to clear. It is in the firm’s interest to have the

check clear as quickly as possible since then the firm can use the money. In

order to speed up this clearing process, firms open offices (called lockboxes)

in different cities to handle the checks.

**PROBLEM :**

We receive payments from 3 regions (South, West and East). The average daily value from each region is as follows:

$360,000 from the South, $600,000 from the West and $720,000 from

the East. We are considering opening lock-

boxes in Los Angeles, Pittsburgh and/or Boston. Operating a

lockbox costs $90,000 per year. The average days from mailing to clearing

is given in Table . Which lockboxes should we open?

From L.A. Pittsburgh Boston

West 2 4 6

East 6 5 2

South 7 5 6

**SOLUTION :**

First we must calculate the lost interest for each possible assignment.

For example, if the West sends its checks to a lockbox in Boston, then on

average there will be $3,600,000 (= 6 × $600, 000) in process on any given

day. Assuming an interest rate of 5%, this corresponds to a yearly loss

of $180,000.

From L.A. Pittsburgh Boston

West 60 120 180

East 216 180 72

South 126 90 108

To formulate the problem as an integer linear program, we will use the

following variables. Let y j be a 0–1 variable that is 1 if lockbox j is opened

and 0 if it is not. Let xij be 1 if region i sends its checks to lockbox j.

The objective is to minimize total yearly costs:

60x11 + 120x12 + 180x13 + 216x21 + . . . + 90y1 + 90y2 + 90y3 .

Each region must be assigned to one lockbox:

Summation of all x ij = 1 for all i.

The regions cannot send checks to closed lockboxes. For lockbox 1 (Los

Angeles), this can be written as:

x11+ x21+x31≤ 3y1 .

Indeed, suppose that we do not open a lockbox in L.A. Then y1 is 0, so all of

x11, x21 and x31 must also be. On the other hand, if we open a lockbox

in L.A., then y1 is 1 and there is no restriction on the x values.

We can create constraints for the other lockboxes to finish off the integer

program. For this problem, we would have 12 variables (3 y variables, 9 x

variables) and 6 constraints. This gives the following integer program:

MIN

60 x11+120 x12 + 180 x13+216 x21+180 x22+72 x23+126x31+90x32+108x33+90 y1 + 90 y2 + 90 y3

SUBJECT TO

x11+x12+x13= 1

x21+x22+x23 =1

x31+x32+x33 =1

x11+x21+x31-3y1<= 0

x12+x22+x32-3y2<=0

x13+x23+x33-3y3<=0

If we ignore integrality, we get the solution x 11 = x 23=x32 = 1,

y 1 =y2= y 3 = 0.33 and the rest equals 0. Note that we get no useful

information out of this linear programming solution: all 3 regions look the

same.

For instance,

consider the nine constraints of the form

x ij ≤ y j .

i.e

x11-y1<=0

x21-y1<=0

x31-y1<=0

x12-y2<=0

x22-y2<=0

x32-y2<=0

x13-y3<=0

x23-y3<=0

x33-y3<=0

These constraints also force a region to only use open lockboxes. It might

seem that a larger formulation is less efficient and therefore should be

avoided. This is not the case! If we solve the linear program with the

above constraints, we get the solution **x 11 = x 23 = x 33 = y 1 = y 3 = 1**

**with the rest equal to zero**. In fact, we have an integer solution, which must

therefore be optimal! Different integer programming formulations can have

very different properties with respect to their linear programming relax-

ations. As a general rule, one prefers an integer programming formulation

whose linear programming relaxation provides a tight bound.